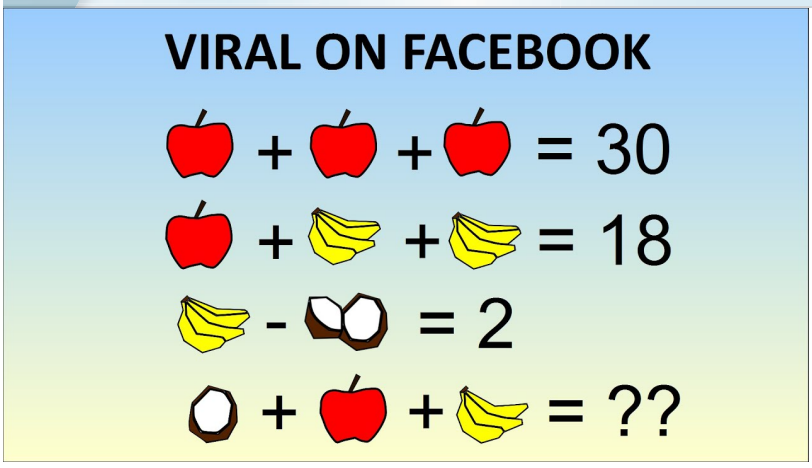
6-2 Solving Systems Using Substitution



$a+a+a=30$ $\quad a+2b=18$ $\quad b-c=2$

$\frac{3a}{3}=\frac{30}{3}$ $\quad 10+2b=18$ $\quad 4-c=2$

$a=10$ $\quad -10 \quad -10$ $\quad -4 \quad -4$

$2b=8$ $\quad -c=-2$

$\frac{2}{2}=\frac{8}{2}$ $\quad c=2$

$b=4$

$\frac{1}{2}c+a+\frac{3}{4}b \Rightarrow \frac{1}{2}(2)+10+\frac{3}{4}(4)$

$1+10+3=14$

The above problem is a linear system. You can solve linear systems by solving one of the equations for one of the variables. Then substitute the expression for the variable into the other equation. This is called the ***substitution method***.

Systems of equations can be solved in more than one way. When a system has at least one equation that can be solved quickly for a variable, the system can solved efficiently using substitution.

SUBSTITUTION METHOD OF SOLVING LINEAR EQUATIONS

1. Isolate one of the variables in one of the equations
2. Substitute the expression for the variable into the other equation
3. Solve that new equation
4. Substitute that solution back into one of the original equations to find the solution for the other variable
5. Check your solution
6. Write your answer as an ordered pair

PROBLEM 1: USING THE SUBSTITUTION METHOD

Solve each system using substitution. Check your solutions.

1. $\begin{cases} y=3x-4 \\ x+y=-32 \end{cases}$

$x+(3x-4)=-32$

$4x-4=-32$

$+4 \quad +4$

$\frac{4x}{4}=\frac{-28}{4}$

$x=-7$

$y=3(-7)-4$

$y=-21-4$

$y=-25$

$(-7,-25)$

2. $\begin{cases} -2x+8y=40 \\ y=6x+28 \end{cases}$

$-2x+8(6x+28)=40$

$-2x+48x+224=40$

$46x+224=40$

$-224 \quad -224$

$\frac{46x}{46}=\frac{-186}{46}$

$x=-4$

$y=6(-4)+28$

$y=-24+28$

$y=4$

$(-4,4)$

3. $\begin{cases} y=4x+3 \\ 8x-y=-7 \end{cases}$

$8x-(4x+3)=-7$

$8x-4x-3=-7$

$4x-3=-7$

$+3 \quad +3$

$\frac{4x}{4}=\frac{-4}{4}$

$x=-1$

$y=4(-1)+3$

$y=-4+3$

$y=-1$

$(-1,-1)$

To use the substitution method to solve a system of equations, one of the equations must be solved for a variable. If none of the equations given are in that form, you must rewrite one of them to isolate one of the variables. Look for the easiest variable to isolate.

PROBLEM 2: SOLVING FOR A VARIABLE AND USING SUBSTITUTION

Solve each system using substitution. Check your solutions.

4. $\begin{cases} 4x + 3y = 14 \\ -2x + y = -3 \end{cases}$

$+2x \quad +2x$

$y = -3 + 2x$

$4x + 3(-3+2x) = 14$

$4x - 9 + 6x = 14$

$10x - 9 = 14$

$+9 \quad +9$

$\frac{10x}{10} = \frac{23}{10}$

$x = 2.3$

$y = -3 + 2(2.3)$

$y = -3 + 4.6$

$y = 1.6$

$(2.3, 1.6)$

5. $\begin{cases} 6y + 5x = -7 \\ 3y - x = -7 \end{cases}$

$-3y \quad -3y$

$-x = -7 - 3y$

$x = 7 + 3y$

$6y + 5(7+3y) = -7$

$6y + 35 + 15y = -7$

$21y + 35 = -7$

$-35 \quad -35$

$\frac{21y}{21} = \frac{-42}{21}$

$y = -2$

$x = 7 + 3(-2)$

$x = 7 - 6$

$x = 1$

$(1, -2)$

6. $\begin{cases} -7m + 5n = 29 \\ \frac{3m}{3} + \frac{9n}{3} = \frac{21}{3} \end{cases}$

$m + 3n = 7$

$-3n \quad -3n$

$m = 7 - 3n$

$-7(7-3n) + 5n = 29$

$-49 + 21n + 5n = 29$

$-49 + 26n = 29$

$+49 \quad +49$

$\frac{26n}{26} = \frac{78}{26}$

$n = 3$

$m = 7 - 3(3)$

$m = 7 - 9$

$m = -2$

$(-2, 3)$

PROBLEM 3: USING SYSTEMS OF EQUATIONS TO SOLVE PROBLEMS

7. A snack bar sells two sizes of snack packs. A large snack pack is $5, and a small snack pack is $3. In one day, the snack bar sold 60 snack packs for a total of $220. How many small snack packs did the snack bar sell?

Let x = small packs

y = large packs

$3x + 5y = 220$

$x + y = 60$

$-y \quad -y$

$x = 60 - y$

$3(60-y) + 5y = 220$

$180 - 3y + 5y = 220$

$180 + 2y = 220$

$-180 \quad -180$

$\frac{2y}{2} = \frac{40}{2}$

$y = 20$

$x = 60 - 20$

$x = 40$

40 small snack packs

8. Your school is planning to bring 193 students to watch the competition cheerleaders perform in Hershey. There are eight drivers available and two types of vehicles school buses and minivans. The buses seat 51 students each, and the minivans seat 8 students each. How many buses and minivans will be needed?

Let b = buses

m = minivans

$\begin{cases} 51b + 8m = 193 \\ b + m = 8 \end{cases}$

$-m \quad -m$

$b = 8 - m$

$51(8-m) + 8m = 193$

$408 - 51m + 8m = 193$

$408 - 43m = 193$

$-408 \quad -408$

$\frac{-43m}{-43} = \frac{-215}{-43}$

$m = 5$

$b = 8 - 5$

$b = 3$

3 buses & 5 minivans

$P = 2l + 2w$

9. A rectangle is 4 times longer than it is wide. The perimeter of the rectangle is 30 cm. Find the dimensions of the rectangle.

Let l = length
w = width

$$\begin{cases} l = 4w \\ 30 = 2l + 2w \end{cases}$$

$30 = 2(4w) + 2w$

$30 = 8w + 2w$

$\frac{30}{10} = \frac{10w}{10}$

$3 = w$

$l = 4(3)$

$l = 12$

12 cm x 3 cm

10. At an ice cream parlor, ice cream cones cost $1.10 and sundaes cost $2.35. One day, the receipts for a total of 172 cones and sundaes were $294.20. How many cones were sold?

Let x = cones
y = sundaes

$$\begin{cases} 1.1x + 2.35y = 294.2 \\ x + y = 172 \end{cases}$$

$-y \quad -y$

$x = 172 - y$

$1.1(172 - y) + 2.35y = 294.2$

$189.2 - 1.1y + 2.35y = 294.2$

$189.2 + 1.25y = 294.2$

$-189.2 \quad -189.2$

$\frac{1.25y}{1.25} = \frac{105}{1.25}$

$y = 84$

$x = 172 - 84$

$x = 88$

88 cones

11. You purchase 8 gal of paint and 3 brushes for $152.50. The next day, you purchase 6 gal of paint and 2 brushes for $113.00. How much does each gallon of paint and each brush cost?

Let x = cost of a gallon of paint
y = cost of a brush

$$\begin{cases} 8x + 3y = 152.5 \\ \frac{6x}{2} + \frac{2y}{2} = \frac{113}{2} \end{cases}$$

$3x + y = 56.5$

$-3x \quad -3x$

$y = 56.5 - 3x$

$8x + 3(56.5 - 3x) = 152.5$

$8x + 169.5 - 9x = 152.5$

$-x + 169.5 = 152.5$

$-169.5 \quad -169.5$

$-x = -17$

$x = 17$

$y = 56.5 - 3(17)$

$y = 56.5 - 51$

$y = 5.5$

$17 per gal of paint

$5.50 per brush

12. You have 26 coins that are made up of nickels and quarters. The value of the coins is $3.10. Find the number of nickels and quarters.

Let n = nickels
q = quarters

$$\begin{cases} 5n + 25q = 310 \\ n + q = 26 \end{cases}$$

$-n \quad -n$

$q = 26 - n$

$5n + 25(26 - n) = 310$

$5n + 650 - 25n = 310$

$-20n + 650 = 310$

$-650 \quad -650$

$\frac{-20n}{-20} = \frac{-340}{-20}$

$n = 17$

$q = 26 - 17$

$q = 9$

17 nickels & 9 quarters

13. Find the value of two numbers if their sum is 149 and their difference is 45.

Let x = a number

y = another number

$x + y = 149$

$x - y = 45$

$+y \quad +y$

$x = 45 + y$

$(45 + y) + y = 149$

$45 + 2y = 149$

$-45 \quad -45$

$\frac{2y}{2} = \frac{104}{2}$

$y = 52$

$x = 45 + 52$

$x = 97$

52 & 97

SPECIAL CASES

If you get an identity, like $2 = 2$, when you solve a system of equations, then the system has *infinitely many solutions*. If you get a false statement, like $8 = 2$, then the system as *no solution.*

PROBLEM 4: SYSTEMS WITH INFINITELY MANY SOLUTIONS OR NO SOLUTION

Solve each system using substitution.

14. $\begin{cases} -2y + 4 = x \\ 3.5x + 7y = 14 \end{cases}$

$3.5(-2y + 4) + 7y = 14$

$-7y + 14 \quad +7y = 14$

$14 = 14$

Infinitely Many Solutions

15. $\begin{cases} y = 3x - 11 \\ y - 3x = -13 \end{cases}$

$(3x - 11) - 3x = -13$

$-11 \neq -13$

No Solution

16. $\begin{cases} 6y + 5x = 8 \\ 2.5x + 3y = 4 \end{cases}$

Lesson Check

Do you know HOW?

Solve each system using substitution. Check your solution.

1. $4y = x$
   $3x - y = 70$

2. $-2x + 5y = 19$
   $3x - 4 = y$

Tell whether the system has *one solution*, *infinitely many solutions*, or *no solution*.

3. $y = 2x + 1$
   $4x - 2y = 6$

4. $-x + \frac{1}{2}y = 13$
   $x + 15 = \frac{1}{2}y$

5. **Talent Show** In a talent show of singing and comedy acts, singing acts are 5 min long and comedy acts are 3 min long. The show has 12 acts and lasts 50 min. How many singing acts and how many comedy acts are in the show?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

6. **Vocabulary** When is the substitution method a better method than graphing for solving a system of linear equations?

fractions/decimals as intercepts or solutions

For each system, tell which equation you would first use to solve for a variable in the first step of the substitution method. Explain your choice.

7. $-2x + y = -1$
   $4x + 2y = 12$

8. $2.5x - 7y = 7.5$
   $6x - y = 1$

Tell whether each statement is *true* or *false*. Explain.

9. When solving a system using substitution, if you obtain an identity, then the system has no solution. F

10. You cannot use substitution to solve a system that does not have a variable with a coefficient of 1 or -1. F

HW due TUE

Name _____ ***6-2 Practice Worksheet*** Period _____

Solve each system using substitution. Check your solution.

1. $m = 5p + 8$
   $m = -10p + 3$

2. $t = 0.2s + 10$
   $4s + 5t = 35$

3. $3x + 5y = 2$
   $x + 4y = -4$

4. The length of a rectangle is 5 cm more than twice the width. The perimeter of the rectangle is 34 cm. Find the dimensions of the rectangle.

5. Suppose you have $28.00 in your bank account and start saving $18.25 every week. Your friend has $161.00 in his account and is withdrawing $15 every week. When will your account balances be the same?

6. A farmer grows only sunflowers and flax on his 240-acre farm. This year he wants to plant 80 more acres of sunflowers than of flax. How many acres of each crop does the farmer need to plant?

7. Suppose you are thinking about buying one of two cars. Car A will cost $17,655. You can expect to pay an average of $1230 per year for fuel, maintenance, and repairs. Car B will cost about $15,900. Fuel, maintenance, and repairs for it will average about $1425 per year. After how many years are the total costs for the cars the same?

8. You have 28 coins that are all nickels and dimes. The value of the coins is $2.05. Find the number of nickels and the number of dimes.

9. You have $3.70 coins that are all dimes and quarters. You have 5 more quarters than dimes. Find the number of dimes and the number of quarters.

10. Michelle and Pam are running a 200-m race. Michelle runs at an average of 7.5 m/s. Pam averages 7.8 m/s, but she starts 1 s after Michelle.

a) How long will it take Pam to catch up to Michelle?
b) Will Pam overtake Michelle before the finish line?

Bell Ringer (from 6-1)

Without graphing, find how many solutions (one, ∞, or none) the system has.

$$\begin{cases} y = -\frac{2}{3}x - 7 \\ 2x + 3y = -21 \end{cases}$$

$m = -\frac{2}{3}$ $b = -7$

$-2x \quad -2x$

$\frac{3y}{3} = \frac{-2x}{3} - \frac{21}{3}$

$y = -\frac{2}{3}x - 7$ $\quad m = -\frac{2}{3}, b = -7$

Infinitely Many Solutions